Heidelberg Collaboratory for Image Processing

The Variational Structure of Disparity and Regularization of 4D Light Fields

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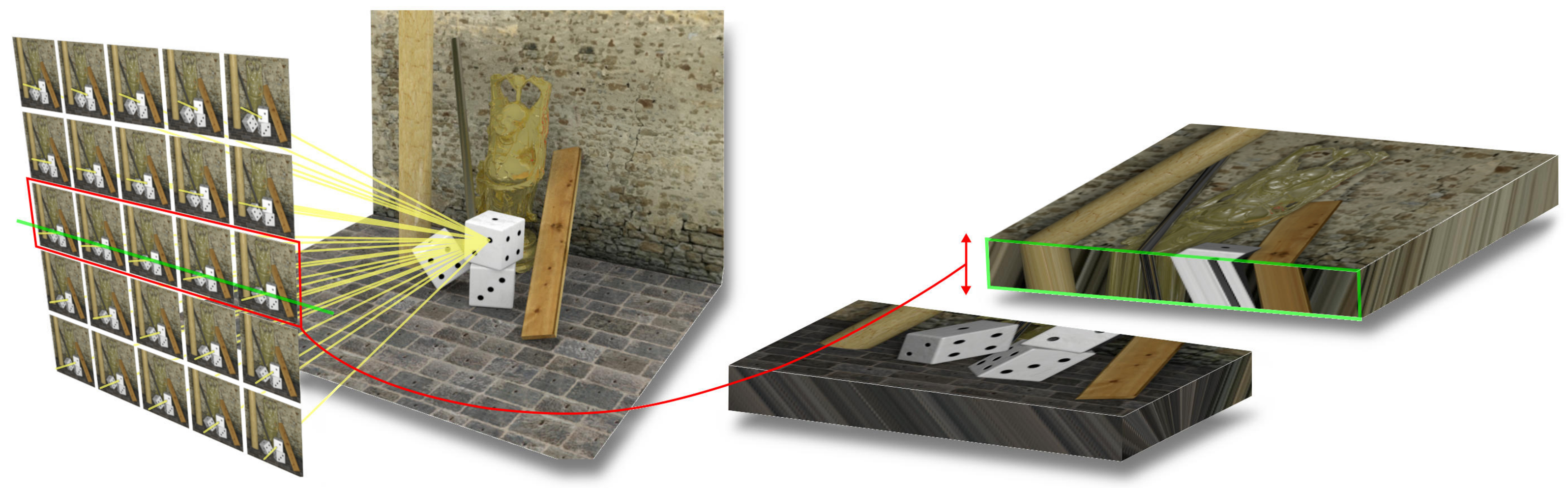
Contributions

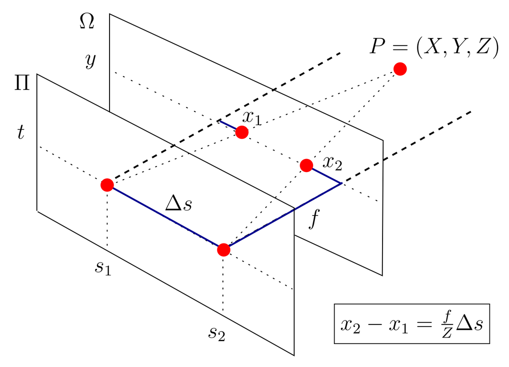
- General variational framework for inverse problems on ray space
- Specialization to arbitrary convex data terms and spatial regularizers possible
- Currently implemented are denoising, disparity map regularization, inpainting, multi-label segmentation

4D Light Field Parametrization and Epipolar Plane Images (EPIs)

Light field structure



Disparity and epipolar plane images



Light field parametrization

Epipolar plane image (EPI)

Disparity equals local slope

General inverse problems on ray space

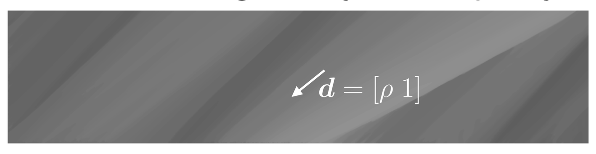
The general inverse problem

Find a vector field $\boldsymbol{U}$ on ray space $\mathcal{R}$ which minimizes

$$\operatorname*{argmin}_{\boldsymbol{U}:\mathcal{R}\to\mathbb{R}^d} \Big\{ \underbrace{J(\boldsymbol{U})}_{\text{convex ray space regularizer}} + \underbrace{F(\boldsymbol{U})}_{\text{convex data term encodes problem}} \Big\}.$$

Regularizer for an epipolar plane image

Regularization needs to be performed in the direction of epipolar lines, which is given by the disparity field ρ:



This can be enforced by using an *anisotropic total variation*

$$J_\rho(\boldsymbol{U}_{y^*,t^*}) := \sum_{i=1}^{n} \int \sqrt{(\nabla U^i_{y^*,t^*})^T D_\rho \nabla U^i_{y^*,t^*}}\, \mathrm{d}(x,s),$$

where the tensor D_ρ encodes the direction information.

Regularizer for ray space

Sum of independent convex regularizers for epipolar plane images in (y,t) and (x,s) coordinates as well as pinhole views in (x,y) coordinates:

$$J_{\lambda\mu}(\boldsymbol{U}) = \mu J_{xs}(\boldsymbol{U}) + \mu J_{yt}(\boldsymbol{U}) + \lambda J_{st}(\boldsymbol{U})$$

$$\text{with } J_{xs}(\boldsymbol{U}) = \int J_\rho(\boldsymbol{U}_{x^*,s^*})\, \mathrm{d}(x^*,s^*)$$

$$J_{yt}(\boldsymbol{U}) = \int J_\rho(\boldsymbol{U}_{y^*,t^*})\, \mathrm{d}(y^*,t^*)$$

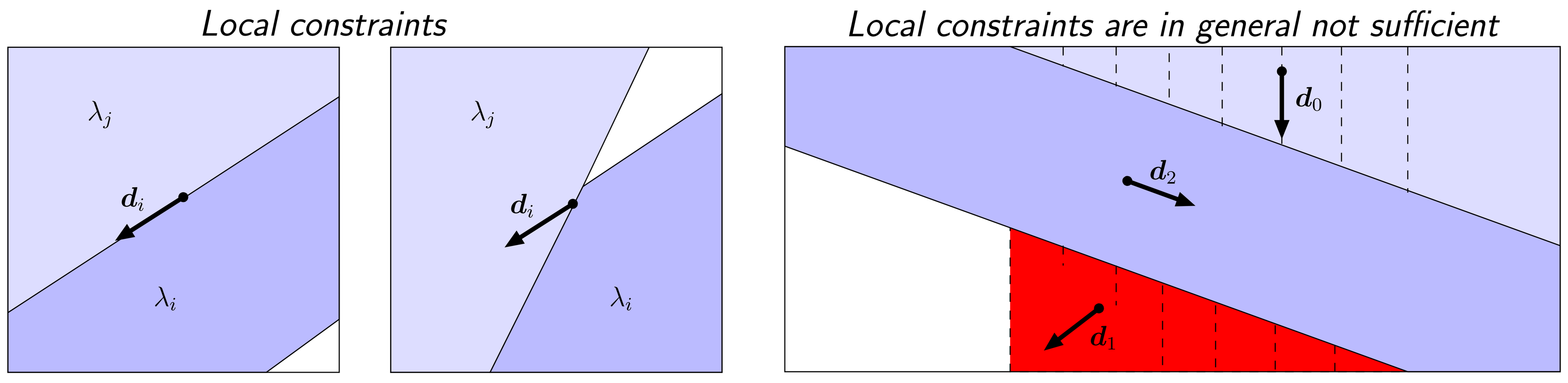
$$J_{st}(\boldsymbol{U}) = \int J_V(\boldsymbol{U}_{s^*,t^*})\, \mathrm{d}(s^*,t^*).$$

Optimization scheme

Constraints on disparity and disparity regularization

Constraints on disparity maps

Local constraints | *Local constraints are in general not sufficient*



Allowed if $\lambda_i < \lambda_j$ — *Forbidden* if $\lambda_i < \lambda_j$ — *Global constraints required, but too costly to optimize*
Unneccessary if baselines are small !

Variational energy for the constraints

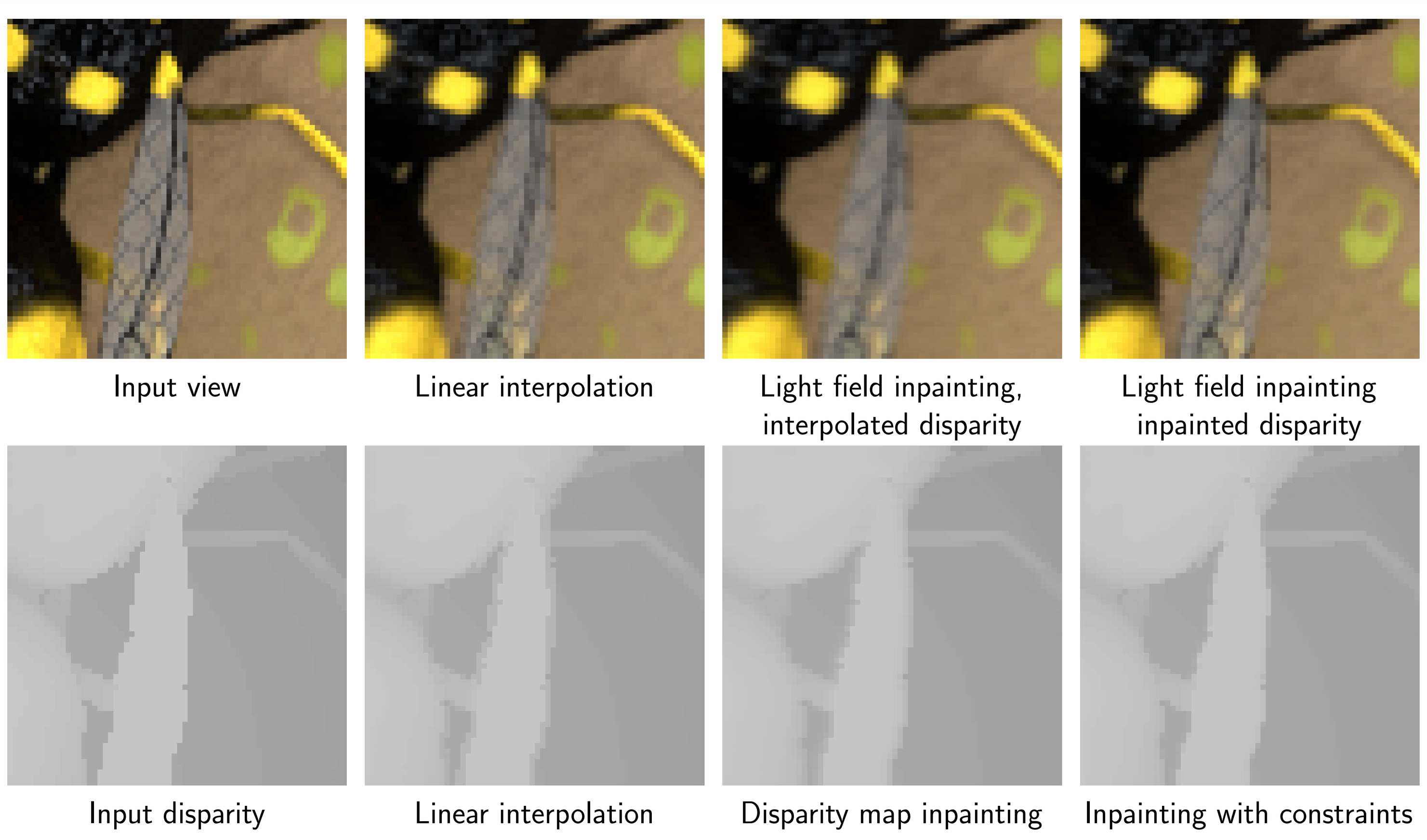
$$E^{\pm\boldsymbol{d}}_{x^*,s^*}(\rho_{x^*,s^*}) = \int \min(\nabla_{\pm\boldsymbol{d}}\, \rho_{x^*,s^*}, 0)^2\, \mathrm{d}(y,t)$$

$$E^{\pm\boldsymbol{d}}_{y^*,t^*}(\rho_{y^*,t^*}) = \int \min(\nabla_{\pm\boldsymbol{d}}\, \rho_{y^*,t^*}, 0)^2\, \mathrm{d}(x,s)$$

Constrained disparity map estimation

Regularization	disparity MSE in pixels $\cdot 10^2$			
	local	single view	rayspace	constrained
Average	4.602	2.727	2.240	1.997

Inpainting for view interpolation



Light field inpainting for view interpolation. Intermediate views in the upsampled light field were marked as unknown regions before solving the inpainting model.

Light field denoising

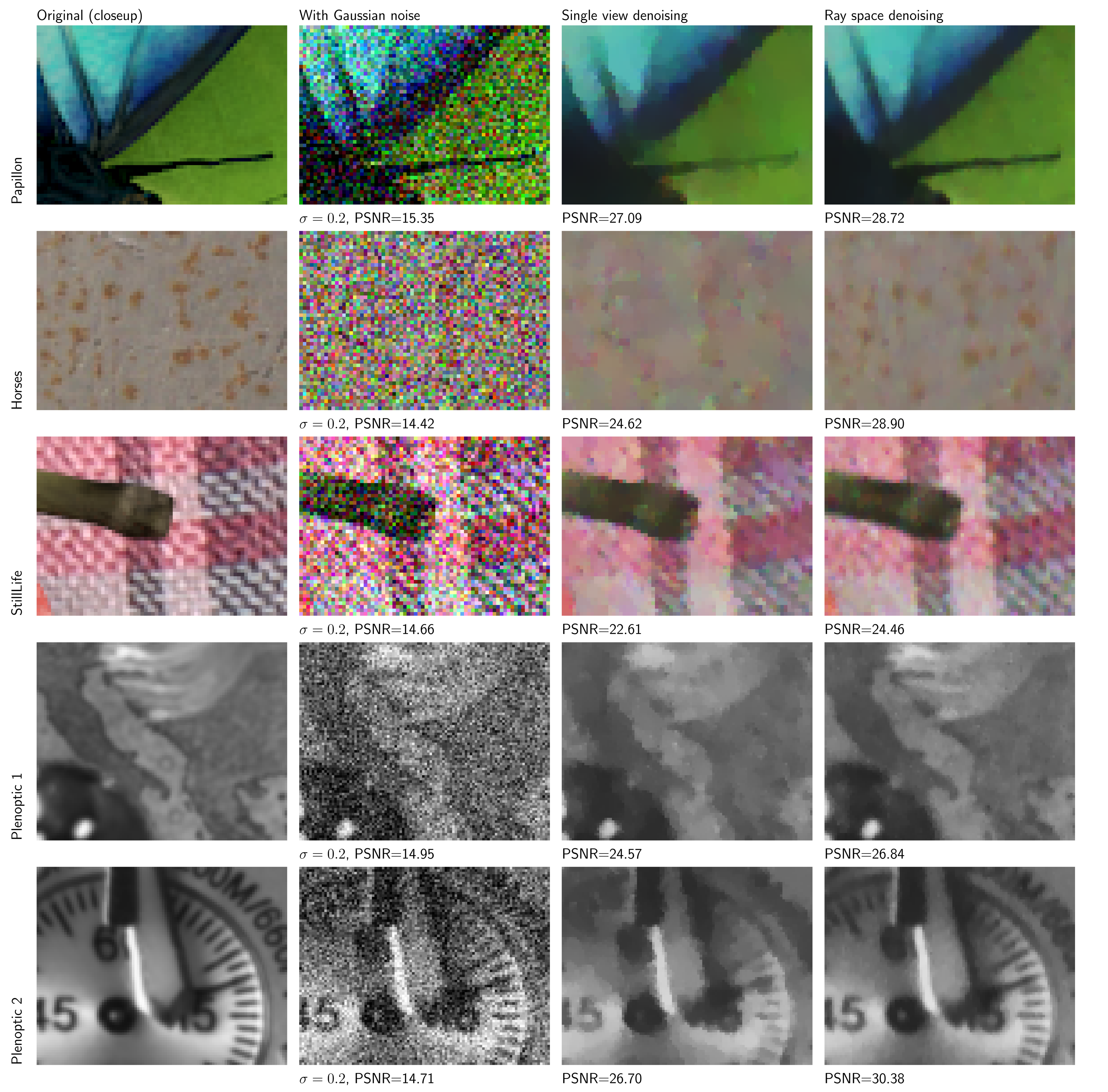
Denoising data term

First experiments were performed with a simple L^2-norm dataterm

$$F(\boldsymbol{U}) = \int_{\mathcal{R}} (\boldsymbol{U} - \boldsymbol{F})^2\, \mathrm{d}(x,y,s,t),$$

where $\boldsymbol{F}$ is the light field to be denoised. Extensions to more complex data terms and spatial regularizers are straight-forward.

Results (total variation regularizer)



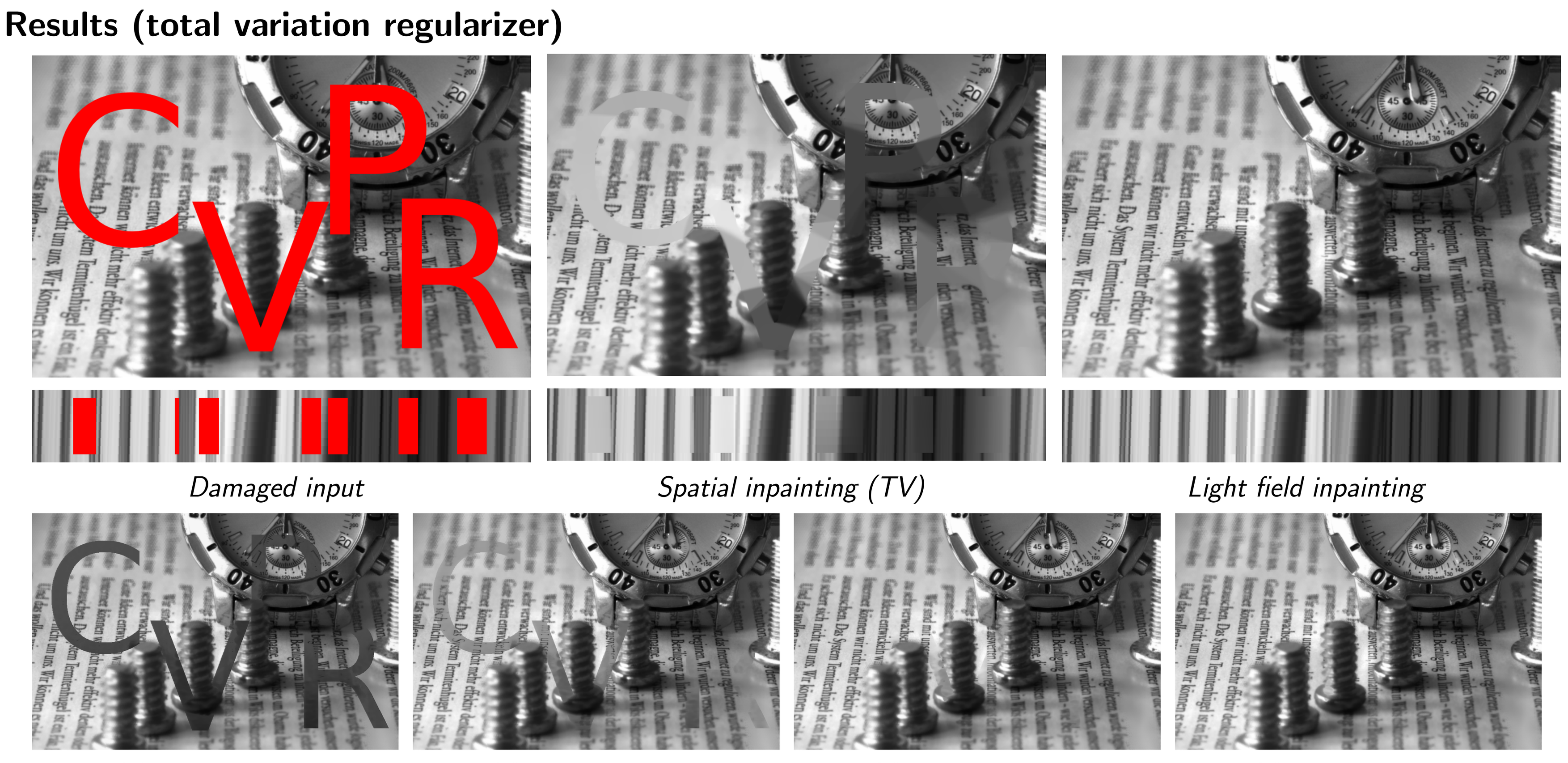
Light field inpainting

Inpainting model

Let $\Gamma \subset \mathcal{R}$ be a region in ray space where the input light field $\boldsymbol{F}$ is unknown. The goal is to recover a function $\boldsymbol{U}$ which restores the missing values. For this, we find

$$\operatorname*{argmin}_{\boldsymbol{U}} J_{\lambda\mu}(\boldsymbol{U}) \text{ such that } \boldsymbol{U} = \boldsymbol{F} \text{ on } \Omega \setminus \Gamma.$$

Results (total variation regularizer)



Optimization

To solve the general **inverse problem on ray space**, we initialize the unknown vector-valued function with $\boldsymbol{U} = 0$ and iterate the following steps:

- data term descent: $\boldsymbol{U} \leftarrow \boldsymbol{U} - \frac{1}{L}\nabla F(\boldsymbol{U})$,
- EPI regularizer descent:

$$\boldsymbol{U}_{x^*,s^*} \leftarrow \mathrm{prox}_{L^{-1}\mu J_\rho}(\boldsymbol{U}_{x^*,s^*}) \text{ for all } (x^*,s^*),$$
$$\boldsymbol{U}_{y^*,t^*} \leftarrow \mathrm{prox}_{L^{-1}\mu J_\rho}(\boldsymbol{U}_{y^*,t^*}) \text{ for all } (y^*,t^*),$$

- spatial regularizer descent:

$$\boldsymbol{U}_{s^*,t^*} \leftarrow \mathrm{prox}_{L^{-1}\lambda J_V}(\boldsymbol{U}_{s^*,t^*}) \text{ for all } (s^*,t^*).$$

All proximity operators above are two-dimensional problems with an L^2 dataterm.

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